

Some useful formulas provided without explanation.
 You are expected to know when and how to use them.

De Moivre's formula: $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$.

The Cauchy–Riemann equations: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.

Some complex functions:

$$e^z = \exp z = e^x(\cos y + i \sin y)$$

$$\ln z = \log z = \ln|z| + i \arg z, \quad \text{Ln } z = \text{Log } z = \ln|z| + i \text{Arg } z$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}, \quad \sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

Cauchy's generalized formula:

$$f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(\zeta)}{(\zeta - z)^{n+1}} d\zeta$$

Some power series:

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n = 1 + z + z^2 + z^3 + \dots$$

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$$

$$\cos z = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} z^{2k} = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \dots$$

$$\sin z = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} z^{2k+1} = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \dots$$

Some trigonometric identities:

$$\begin{aligned} \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v & \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \sin 2u &= 2 \sin u \cos u & \cos 2u &= \cos^2 u - \sin^2 u \\ & & &= 2 \cos^2 u - 1 \\ & & &= 1 - 2 \sin^2 u \end{aligned}$$

$$\begin{aligned} 2 \sin u \cos v &= \sin(u - v) + \sin(u + v) & 2 \cos u \cos v &= \cos(u - v) + \cos(u + v) \\ 2 \sin u \sin v & & 2 \sin u \sin v &= \cos(u - v) - \cos(u + v) \end{aligned}$$

Fourier series for a periodic function with period $2L$:

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/L} = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

$$c_n = \frac{1}{2L} \int_{-L}^L f(x) e^{-in\pi x/L} dx,$$

$$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx, \quad a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx,$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx.$$

Cosine and sine series for a function defined on $[0, L]$:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}, \quad a_0 = \frac{1}{L} \int_0^L f(x) dx,$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx,$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}, \quad b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx.$$

Some integrals:

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

$$\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C$$

$$\int x^n (\ln x)^m dx = \frac{x^{n+1}}{n+1} (\ln x)^m - \frac{m}{n+1} \int x^n (\ln x)^{m-1} dx$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$$

$$\int x^m \cos bx dx = \frac{x^m \sin bx}{b} - \frac{m}{b} \int x^{m-1} \sin bx dx$$

$$\int x^m \sin bx dx = -\frac{x^m \cos bx}{b} + \frac{m}{b} \int x^{m-1} \cos bx dx$$