75032 Matrisemetoder 26. mai 1995: Fasit

1a

$$A = LU = \begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ 1 & 2 & 1 & \\ 1 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ & 1 & 2 & 3 \\ & & 1 & 3 \\ & & & 1 \end{bmatrix} = LL^{\mathrm{T}}, \qquad x = \begin{bmatrix} -5 \\ 18 \\ -20 \\ 8 \end{bmatrix}$$

1b Determinanten er 1.

2a Basis for $\mathcal{N}(A)$ for eksempel:

$$\text{For } \mathcal{N}(A)\text{:} \quad \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -7 \\ 0 \\ 1 \\ 1 \end{bmatrix} \qquad \text{for } \mathcal{R}(A^{\mathrm{T}})\text{:} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

2b $\mathcal{R}(A) = \mathcal{N}(A)$ og $\mathcal{N}(A^{\mathrm{T}}) = \mathcal{R}(A^{\mathrm{T}})$. Bruk basisene fra punkt (a).

2c

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3a

$$A = Q_1 \Sigma Q_2^{\mathrm{T}}; \qquad Q_1 = \begin{bmatrix} 2/3 & -1/\sqrt{2} & 1/(3\sqrt{2}) \\ 2/3 & 1/\sqrt{2} & 1/(3\sqrt{2}) \\ 1/3 & 0 & -4/(3\sqrt{2}) \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 15 & 0 \\ 0 & 5\sqrt{2} \\ 0 & 0 \end{bmatrix}, \quad Q_2 = \frac{1}{5}\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$$

3b

$$x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

4a

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ -1 & 1 & 0 & 0 & \cdots \\ 1 & -1 & 1 & 0 & \cdots \\ -1 & 1 & -1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & -1 & \cdots \\ 0 & 1 & -1 & 1 & \cdots \\ 0 & 0 & 1 & -1 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

4b

$$\begin{bmatrix} 2 & 1 & & & \\ 1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2 & 1 \\ & & & 1 & 1 \end{bmatrix}.$$

5 $(1 - r^2)^{n-1}$, $|r| < 1$.