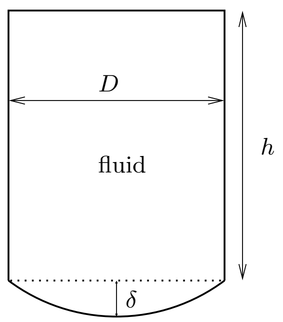
Exercise Set 1

Problem 1



The relevant physical quantities in this problem are the geometry of the vessel (parametrized by d, D and δ), the density ρ and the height h of the liquid which determine the action of the gravity and the modulus of elasticity E which gives a relation between stress and deformation (it relates somehow the pressure and δ).

We sum up in the following table the units of all these quantities

	δ	D	h	d	ρ	g	E
kg	0	0	0	0	1	0	1
m	1	1	1	1	-3	1	-1
s	0	0	0	0	0	-2	-2

The columns given by D, ρ and g are independant. We take these variables as reference variables and we get 7-3=4 independant dimensionless variables:

$$\Pi_1 = \frac{\delta}{D},\ \Pi_2 = \frac{h}{D},\ \Pi_3 = \frac{d}{D},\ \Pi_4 = \frac{E}{D\rho g}$$

The Buckingham's pi theorem tells us that there exists a function Φ such that

$$\Pi_1 = \Phi(\Pi_2, \Pi_3, \Pi_4)$$

i.e.

$$\frac{\delta}{D} = \Phi\left(\frac{h}{D}, \frac{d}{D}, \frac{E}{D\rho g}\right) \tag{1}$$

In fact, the liquid deforms the vessel only through the pressure it exerts at the bottom (which is equal to $\rho g h$). As a consequence $\Pi_2 = \frac{h}{D}$ and $\Pi_4 = \frac{E}{D\rho g}$ are not independant and we combine them to get the pressure term ρgh explicitly ($\Pi_{new} = \frac{\Pi_4}{\Pi_2} = \frac{E}{\rho gh}$). Hence, we rewrite (1) as

$$\frac{\delta}{D} = \Phi\left(\frac{E}{\rho gh}, \frac{d}{D}\right)$$

We can obtain the same result in a more rigorous way by starting the dimension analysis again. The relevant physical quantities are now δ,D,d,E and $P = \rho gh$.

	δ	D	d	E	P
kg	0	0	0	1	1
m	1	1	1	-1	-1
s	0	0	0	-2	-2

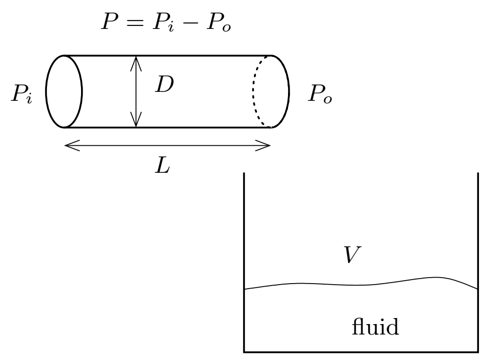
The rank of the system is now 2. We take P and D as reference variables and get 5-2=3 independant variables, namely

$$\Pi_1 = \frac{\delta}{D},\ \Pi_2 = \frac{d}{D},\ \Pi_3 = \frac{E}{P} = \frac{E}{\rho gh}$$

The Buckingham's pi theorem gives us directly that

$$\frac{\delta}{D} = \Phi\left(\frac{E}{\rho gh}, \frac{d}{D}\right)$$

Problem 2



The time required to fill in the vessel depends directly on the flow coming out of the pipe. The flow depends on the diameter of the pipe (D) and on the velocity of the liquid. The striving force in this experiment is the pressure (we assume gravity is not involved) which act through its gradient. Therefore $P = P_i - P_o$ and L must be considered as relevant variables. The viscosity μ

which determines the fluid response to a given excitation must be taken into consideration.

We have

	D	L	P	V	t	μ
kg	0	0	1	0	0	1
m	1	1	-1	3	0	-1
s	0	0	-2	0	1	-1

The rank of the system is 3. We take V, t and μ as reference variables. We have 6-3=3 independant variables. Let's take in details the first one.

$$\Pi_1 = \frac{P}{V^{x_1} t^{x_2} \mu^{x_3}}$$

where x_1, x_2, x_3 are solutions of

$$\begin{pmatrix} 0 & 0 & 1 \\ 3 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

We solve this system and get:

$$\Pi_1 = \frac{Pt}{\mu}$$

similarly, we have:

$$\Pi_2 = \frac{D}{V^{1/3}}, \ \Pi_3 = \frac{L}{V^{1/3}}$$

The buckingham's pi theorem gives us

$$\frac{Pt}{\mu} = \Phi\left(\frac{D}{V^{1/3}}, \frac{L}{V^{1/3}}\right)$$

Φ depends only on the geometries of the pipe and the vessel which remain unchanged during all the experiments. Therefore

$$\frac{Pt}{\mu} = Constant$$

hence

$$\log(P) = \log(\frac{1}{t}) + \log(\mu) \tag{2}$$

which is exactly what give the graphs. Indeed, in the first graph, we have almost straight lines of slope 1 which only differ by their horizontal position which is determined by μ in (2).