TMA4195 Mathematical modelling 2004

Exercise set 1

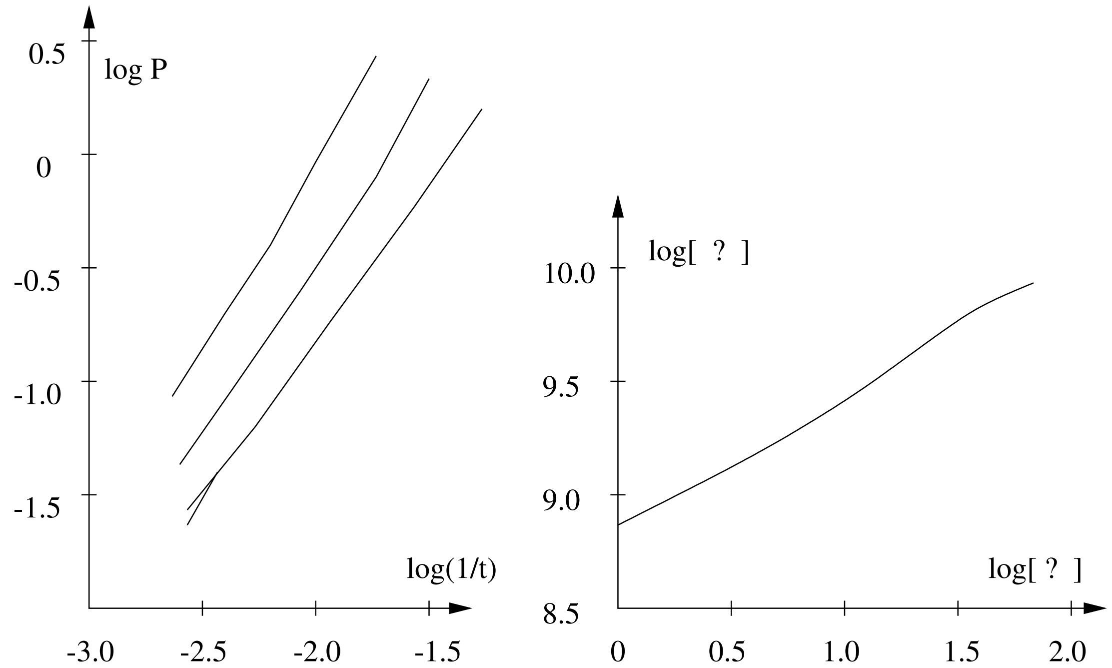
Advice and suggestions: 2004–08–24

Exercise 1: An open, vertical, cylindrical tank with diameter D is filled with a liquid of density ρ to a height h. The bottom has thickness d and (Young's) modulus of elasticity[1] E. The weight of the liquid causes the bottom of the tank to sag a bit, most in the middle (we assume that the edges of the bottom stay put). Show that the amount of sagging δ in the middle can be expressed as

$$\frac{\delta}{D} = \Phi\Big(\frac{h}{D}, \frac{d}{D}, \frac{E}{Dg\rho}\Big).$$

How can you improve on this result with a small bit of physical insight? (*Hint*: Does the liquid influence the bottom other than through its pressure?)

Exercise 2: By measuring the pressure drop p in the feed pipe against the time t taken to fill a vessel of volume V, BOSE, BOSE and RUERT (around 1910) plotted the relations shown in the lefthand figure for water, chloroform, boroform,[2] and mercury. Show by dimensional analysis (using the density ρ and the viscosity[3] μ) that there should be one common relation that turns these curves into one. That is, find the variables along the axes ofVON KÁRMAN's representation of the same data, as shown on the righthand figure.



[1] Young's modulus of elasticity has the dimension of a force per area.

[2] No, I don't know what boroform is, and I cannot find it in any dictionary or encyclopaedia. Perhaps because it is really a Norwegian word, and should be called something else in English. But that is unimportant for our purposes here.

[3] Dynamic viscosity μ has dimensions which can be derived from its definition: A *stress* is a force per area (of which pressure is an example), and shear stresses in a fluid are proportional to the velocity gradient (units: velocity per length). The dynamic viscosity μ is the constant of proportionality.