TMA4195 Mathematical modelling 2004

Exercise set 3

Advice and suggestions: 2004–09–07

Exercise 1: (Exam January 1999) Important parameters in the assessment of foods like meat and fish are water and fat content. For use in quality control there is a need for instruments which quickly and reliably can yield an approximate value for these parameters. In particular, it is an advantage for the test to be non-destructive, so that foods can be tested and then sold. One suggested instrument uses the fact that water, fat and proteins have different thermal properties. The idea is to add a known amount of energy in the form of heat to the surface of the food, and then to measure the resulting increase in the temperature. For this instrument to be useful, it should satisfy the following two criteria:

1. The measurement should not take much more than a half minute.
2. The measurement should give a representative average value for water and fat content not only near the surface, but down to a centimeter or deeper.

Approximate values for the specific capacity of heat and heat conductivity for the most important parts of food are:

			Water	Fat	Protein
Heat capacity	c	$\mathrm{J\,kg^{-1}\,K^{-1}}$	4180	2000	1300
Heat conductivity	k	$\mathrm{J\,m^{-1}\,K^{-1}\,s^{-1}}$	0.56	0.09	0.16

You can assume the density to be 1000 $\mathrm{kg/m^3}$ for all three. Employ a simple dimensional analysis technique to assess whether it would seem probable that such an instrument could be made to fulfill the requirements.

Exercise 2: (Lin & Segel side 243, problem 12)
Find approximate solutions to

$$36x^3 + (162 + 4\varepsilon)x^2 - 24\varepsilon x - 9\varepsilon = 0$$

valid for $|\varepsilon| \ll 1$. (Hint: First find a solution of the form $x_0 + \varepsilon x_1 + \cdots$. There are two more solutions, both of them near 0. Try to rescale the equation by balancing two terms.)

Exercise 3: Find inner and outer approximations (to lowest order) for the solution of the problem

$$\varepsilon y'' + (1 + x^2)y' + y = 0, \qquad y(0) = 0, \quad y(1) = 1.$$

Assume that ε is small but positive, and expect a boundary layer at $x = 0$.